5-2 Solving Quadratic Equations by Graphing

Main Ideas

- Solve quadratic equations by graphing.
- Estimate solutions of quadratic equations by graphing.

New Vocabulary

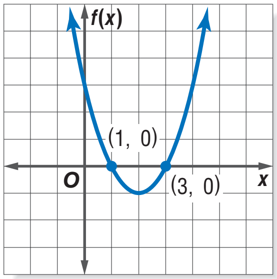
quadratic equation
standard form
root
zero

GET READY for the Lesson

As you speed to the top of a free-fall ride, you are pressed against your seat so that you feel like you're being pushed downward. Then as you free-fall, you fall at the same rate as your seat. Without the force of your seat pressing on you, you *feel* weightless. The height above the ground (in feet) of an object in free-fall can be determined by the quadratic function $h(t) = -16t^2 + h_0$, where t is the time in seconds and the initial height is h_0 feet.

Solve Quadratic Equations When a quadratic function is set equal to a value, the result is a quadratic equation. A **quadratic equation** can be written in the form $ax^2 + bx + c = 0$, where $a \neq 0$. When a quadratic equation is written in this way, and a, b, and c are all integers, it is in **standard form**.

The solutions of a quadratic equation are called the **roots** of the equation. One method for finding the roots of a quadratic equation is to find the **zeros** of the related quadratic function. The zeros of the function are the x-intercepts of its graph. These are the solutions of the related equation because $f(x) = 0$ at those points. The zeros of the function graphed at the right are 1 and 3.



Reading Math

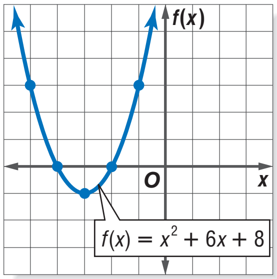
Roots, Zeros, Intercepts In general, equations have roots, functions have zeros, and graphs of functions have x-intercepts.

EXAMPLE 1 Two Real Solutions

Solve $x^2 + 6x + 8 = 0$ by graphing.

Graph the related quadratic function $f(x) = x^2 + 6x + 8$. The equation of the axis of symmetry is $x = -\frac{6}{2(1)}$ or -3. Make a table using x values around -3. Then, graph each point.

x	-5	-4	-3	-2	-1
$f(x)$	3	0	-1	0	3



We can see that the zeros of the function are -4 and -2. Therefore, the solutions of the equation are -4 and -2.

CHECK Your Progress **Solve each equation by graphing.**

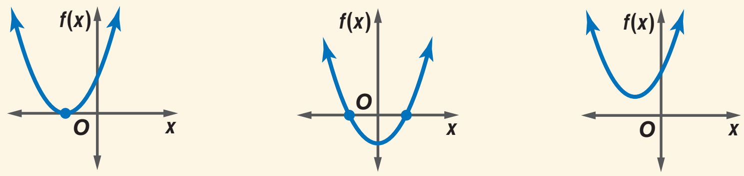
1A. $x^2 - x - 6 = 0$ **1B.** $x^2 + x = 2$

There are three possible outcomes when solving a quadratic equation.

KEY CONCEPT — Solutions of a Quadratic Equation

Words A quadratic equation can have one real solution, two real solutions, or no real solution.

Models One Real Solution — Two Real Solutions — No Real Solution



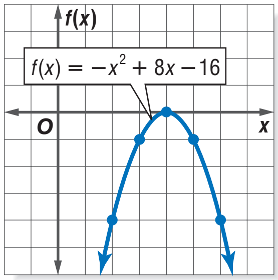
EXAMPLE 2 One Real Solution

Solve $8x - x^2 = 16$ by graphing.

$8x - x^2 = 16 \rightarrow -x^2 + 8x - 16 = 0$ Subtract 16 from each side.

Graph the related quadratic function $f(x) = -x^2 + 8x - 16$.

x	2	3	4	5	6
f(x)	−4	−1	0	−1	−4



Notice that the graph has only one x-intercept, 4. Thus, the equation's only solution is 4.

CHECK Your Progress Solve each equation by graphing.

2A. $10x = -25 - x^2$ **2B.** $-x^2 - 2x = 1$

Study Tip

One Real Solution
When a quadratic equation has one real solution, it really has two solutions that are the same number.

EXAMPLE 3 No Real Solution

NUMBER THEORY Find two real numbers with a sum of 6 and a product of 10 or show that no such numbers exist.

Explore Let x = one of the numbers. Then $6 - x$ = the other number.

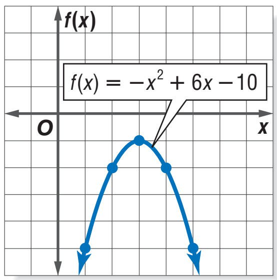
Plan

$x(6 - x) = 10$ The product is 10.

$6x - x^2 = 10$ Distributive Property

$-x^2 + 6x - 10 = 0$ Subtract 10 from each side.

Solve Graph the related function.



The graph has no x-intercepts. This means the original equation has no real solution. Thus, it is *not* possible for two numbers to have a sum of 6 and a product of 10.

Check Try finding the product of several pairs of numbers with sums of 6. Is each product less than 10 as the graph suggests?

CHECK Your Progress

3. Find two real numbers with a sum of 8 and a product of 12 or show that no such numbers exist.

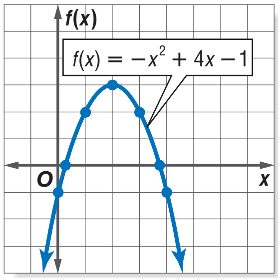
Online Personal Tutor at algebra2.com

Estimate Solutions Often exact roots cannot be found by graphing. You can estimate solutions by stating the integers between which the roots are located.

EXAMPLE 4 Estimate Roots

Solve $-x^2 + 4x - 1 = 0$ by graphing. If exact roots cannot be found, state the consecutive integers between which the roots are located.

x	0	1	2	3	4
$f(x)$	−1	2	3	2	−1



The x-intercepts of the graph indicate that one solution is between 0 and 1, and the other is between 3 and 4.

Study Tip

Location of Roots

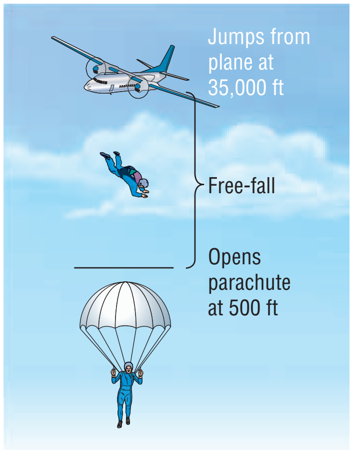
Notice in the table of values that the value of the function changes from negative to positive between the x-values of 0 and 1, and 3 and 4.

CHECK Your Progress

4. Solve $x^2 + 5x - 2 = 0$ by graphing. If exact roots cannot be found, state the consecutive integers between which the roots are located.

Real-World EXAMPLE 5

EXTREME SPORTS **In 1999, Adrian Nicholas broke the world record for the longest human flight. He flew 10 miles from a drop point in 4 minutes 55 seconds using an aerodynamic suit. Using the information at the right and ignoring air resistance, how long would he have been in free-fall had he not used this suit? Use the formula $h(t) = -16t^2 + h_0$, where the time t is in seconds and the initial height h_0 is in feet.**



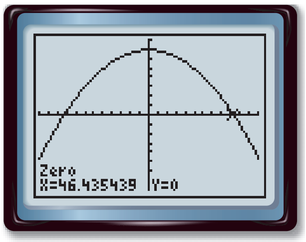
We need to find t when $h_0 = 35{,}000$ and $h(t) = 500$. Solve $500 = -16t^2 + 35{,}000$.

$500 = -16t^2 + 35{,}000$ Original equation

$0 = -16t^2 + 34{,}500$ Subtract 500 from each side.

Graph the related function $y = -16t^2 + 34{,}500$ on a graphing calculator.

Use the **Zero** feature, [2nd] [CALC], to find the positive zero of the function, since time cannot be negative. Use the arrow keys to locate a left bound and press [ENTER]. Then, locate a right bound and press [ENTER] twice. The positive zero of the function is approximately 46.4. Mr. Nicholas would have been in free-fall for about 46 seconds.



[−60, 60] scl: 5 by
[−40000, 40000] scl: 5000

CHECK Your Progress

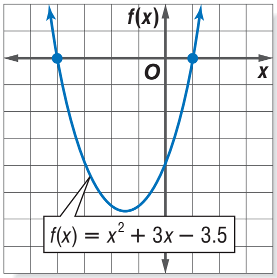
5. If Mr. Nicholas had jumped from the plane at 40,000 feet, how long would he have been in free-fall had he not used his special suit?

CHECK Your Understanding

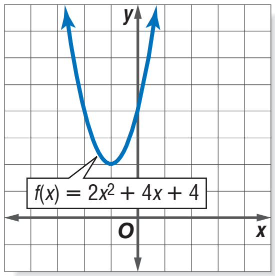
Examples 1–3
(pp. 246–247)

Use the related graph of each equation to determine its solutions.

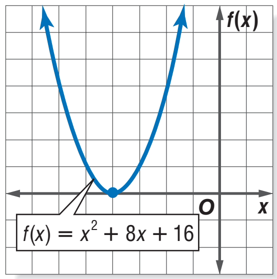
1. $x^2 + 3x - 3.5 = 0$



2. $2x^2 + 4x + 4 = 0$



3. $x^2 + 8x + 16 = 0$



Examples 1–4
(pp. 246–248)

Solve each equation by graphing. If exact roots cannot be found, state the consecutive integers between which the roots are located.

4. $-x^2 - 7x = 0$

5. $x^2 - 2x - 24 = 0$

6. $25 + x^2 + 10x = 0$

7. $-14x + x^2 + 49 = 0$

8. $x^2 + 16x + 64 = -6$

9. $x^2 - 12x = -37$

10. $4x^2 - 7x - 15 = 0$

11. $2x^2 - 2x - 3 = 0$

Examples 1, 3
(pp. 246, 247)

12. NUMBER THEORY Use a quadratic equation to find two real numbers with a sum of 5 and a product of −14, or show that no such numbers exist.

Example 5
(p. 248)

13. ARCHERY An arrow is shot upward with a velocity of 64 feet per second. Ignoring the height of the archer, how long after the arrow is released does it hit the ground? Use the formula $h(t) = v_0t - 16t^2$, where $h(t)$ is the height of an object in feet, v_0 is the object's initial velocity in feet per second, and t is the time in seconds.

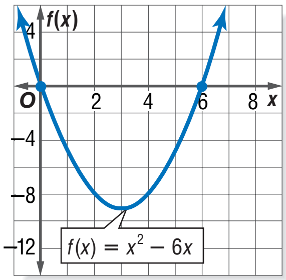
Exercises

HOMEWORK HELP

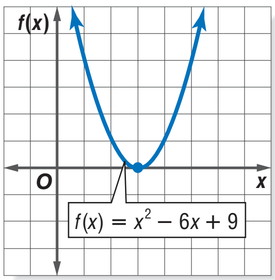
For Exercises	See Examples
14–19	1–3
20–29	1–4
30, 31	5

Use the related graph of each equation to determine its solutions.

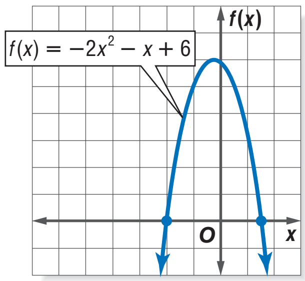
14. $x^2 - 6x = 0$



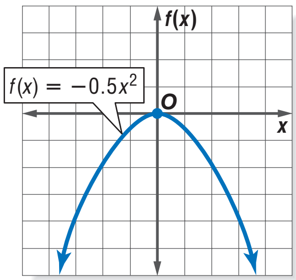
15. $x^2 - 6x + 9 = 0$



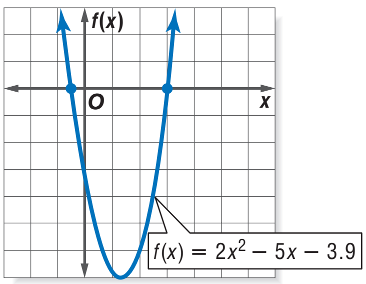
16. $-2x^2 - x + 6 = 0$



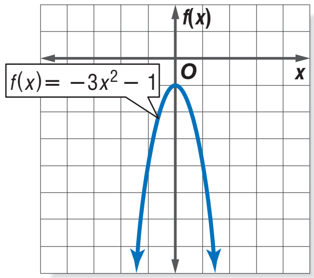
17. $-0.5x^2 = 0$



18. $2x^2 - 5x - 3.9 = 0$



19. $-3x^2 - 1 = 0$



Solve each equation by graphing. If exact roots cannot be found, state the consecutive integers between which the roots are located.

20. $x^2 - 3x = 0$

21. $-x^2 + 4x = 0$

22. $-x^2 + x = -20$

23. $x^2 - 9x = -18$

24. $14x + x^2 + 49 = 0$

25. $-12x + x^2 = -36$

26. $x^2 + 2x + 5 = 0$

27. $-x^2 + 4x - 6 = 0$

28. $x^2 + 4x - 4 = 0$

29. $x^2 - 2x - 1 = 0$

For Exercises 30 and 31, use the formula $h(t) = v_0t - 16t^2$, where $h(t)$ is the height of an object in feet, v_0 is the object's initial velocity in feet per second, and t is the time in seconds.

30. TENNIS A tennis ball is hit upward with a velocity of 48 feet per second. Ignoring the height of the tennis player, how long does it take for the ball to fall to the ground?

31. BOATING A boat in distress launches a flare straight up with a velocity of 190 feet per second. Ignoring the height of the boat, how many seconds will it take for the flare to hit the water?

Real-World Link

Located on the 86th floor, 1050 feet (320 meters) above the streets of New York City, the Observatory offers panoramic views from within a glass-enclosed pavilion and from the surrounding open-air promenade.

Source: www.esbnyc.com

Solve each equation by graphing. If exact roots cannot be found, state the consecutive integers between which the roots are located.

32. $2x^2 - 3x = 9$

33. $4x^2 - 8x = 5$

34. $2x^2 = -5x + 12$

35. $2x^2 = x + 15$

36. $x^2 + 3x - 2 = 0$

37. $x^2 - 4x + 2 = 0$

38. $-2x^2 + 3x + 3 = 0$

39. $0.5x^2 - 3 = 0$

NUMBER THEORY **Use a quadratic equation to find two real numbers that satisfy each situation, or show that no such numbers exist.**

40. Their sum is -17 and their product is 72.

41. Their sum is 7 and their product is 14.

42. Their sum is -9 and their product is 24.

43. Their sum is 12 and their product is -28.

44. LAW ENFORCEMENT Police officers can use the length of skid marks to help determine the speed of a vehicle before the brakes were applied. If the skid marks are on dry concrete, the formula $\frac{s^2}{24} = d$ can be used. In the formula, s represents the speed in miles per hour and d represents the length of the skid marks in feet. If the length of the skid marks on dry concrete are 50 feet, how fast was the car traveling?

45. PHYSICS Suppose you could drop a small object from the Observatory of the Empire State Building. How long would it take for the object to reach the ground, assuming there is no air resistance? Use the information at the left and the formula $h(t) = -16t^2 + h_0$, where t is the time in seconds and the initial height h_0 is in feet.

EXTRA PRACTICE

See pages 900, 930.

Math Online

Self-Check Quiz at algebra2.com

H.O.T. Problems

46. OPEN ENDED Give an example of a quadratic equation with a double root, and state the relationship between the double root and the graph of the related function.

47. REASONING Explain how you can estimate the solutions of a quadratic equation by examining the graph of its related function.

Yagi Studio/SuperStock

48. CHALLENGE A quadratic function has values $f(-4) = -11$, $f(-2) = 9$, and $f(0) = 5$. Between which two x-values must $f(x)$ have a zero? Explain your reasoning.

49. *Writing in Math* Use the information on page 246 to explain how a quadratic function models a free-fall ride. Include a graph showing the height at any given time of a free-fall ride that lifts riders to a height of 185 feet and an explanation of how to use this graph to estimate how long the riders would be in free-fall if the ride were allowed to hit the ground before stopping.

STANDARDIZED TEST PRACTICE

50. ACT/SAT If one of the roots of the equation $x^2 + kx - 12 = 0$ is 4, what is the value of k?

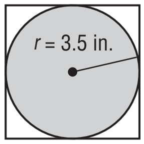
A -1

B 0

C 1

D 3

51. REVIEW What is the area of the square in square inches?



F 49

G 51

H 53

J 55

Spiral Review

Find the y-intercept, the equation of the axis of symmetry, and the x-coordinate of the vertex for each quadratic function. Then graph the function by making a table of values. (Lesson 5-1)

52. $f(x) = x^2 - 6x + 4$

53. $f(x) = -4x^2 + 8x - 1$

54. $f(x) = \frac{1}{4}x^2 + 3x + 4$

55. Solve the system $4x - y = 0$, $2x + 3y = 14$ by using inverse matrices. (Lesson 4-8)

Evaluate the determinant of each matrix. (Lesson 4-3)

56. $\begin{bmatrix} 6 & 4 \\ -3 & 2 \end{bmatrix}$

57. $\begin{bmatrix} 2 & -1 & -6 \\ 5 & 0 & 3 \\ -3 & 2 & 11 \end{bmatrix}$

58. $\begin{bmatrix} 6 & 5 & 2 \\ -3 & 0 & -6 \\ 1 & 4 & 2 \end{bmatrix}$

59. COMMUNITY SERVICE A drug awareness program is being presented at a theater that seats 300 people. Proceeds will be donated to a local drug information center. If every two adults must bring at least one student, what is the maximum amount of money that can be raised? (Lesson 3-4)

GET READY for the Next Lesson

PREREQUISITE SKILL Factor completely. (p. 753)

60. $x^2 + 5x$

61. $x^2 - 100$

62. $x^2 - 11x + 28$

63. $x^2 - 18x + 81$

64. $3x^2 + 8x + 4$

65. $6x^2 - 14x - 12$